Self-Organized Learning of 3 Dimensions

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***Abstract*— Geometry learning capabilities of a competitive neural network is studied. It is shown, that the appropriate selection of neural activity function enables the learning of the 3 dimensional geometry of a world, from two of 2 dimensional projections of 3 dimensional extended objects.**

I. Introduction

The system to be described is an application of the artificial neural network architecture we developed for 2 dimensional (2D) images [7]. Here two of 2D projections of 3 dimensional (3D) objects guide the network to wire in the 3D geometry of the external world. The basis of the system is a self-organizing competitive artificial neural network [4] that receives, as inputs, images of the external world. The primary building block of the algorithm is a 'winner-take-all' network. In such a model every neuron receives every input through its connections or input filter system; this filter system may differ and change during learning. Elements of this filter system shall be called as feedforward connections, denoted by w_{ij}, where indices i and j correspond to the input and the neuron they connect, respectively. The other set of connections that may change during the course of learning are the connections between neurons. This set shall be called as lateral or feedback connections, denoted by q_{kl} $(k, l = 1, 2, \ldots, m)$ and indexed by the two neurons they connect: the first index being the index of the neuron that is sending its output and the other index being the index of the neuron that is receiving the output as an input. The change that these systems go through was called learning and it is given by connection update rules. The update rules could be in the form of a numerical algorithm suitable for software implementation. In other cases the update rules are formulated in the form of differential equations that either model realistic neurons [1, 2, 5] or could be models of analog hardware [3]. In the following the update rules shall be given in the form that is suitable for software implementation.

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II. Forming spatial filters

The numerical procedure of the 'winner-take-all' network is implemented in the following way: first an input is presented to the network and then neurons process their inputs and develop activities in accordance with the equation

$$a_j = F(\mathbf{x}, \mathbf{w}_i)$$

where $\mathbf{x}$ denotes the n dimensional input vector, a_j the input activity of the j^{th} neuron and F is a similarity function Usually $F(\mathbf{x}, \mathbf{w}) = \mathbf{x}\mathbf{w}$, where $\mathbf{x}\mathbf{w}$ denotes the dot inner products of vectors $\mathbf{x}$ and $\mathbf{w}$.

Competition starts. The winner of the competition is the neuron of largest activity. The stored vector of the winning neuron l is then modified with the help of the update rule:

$$\Delta \mathbf{w}_l = \alpha(\mathbf{x} - \mathbf{w}_l)$$

where α is the so called feedforward learning parameter; $0 < \alpha < 1$.

In earlier simulations, we presented 2D objects of a 2D space to the network [7]. Input vectors were derived by computing the overlap of the local, extended, randomly positioned objects and the pixels of digitization. The inputted local, extended objects as well as the 'winner-take-all' mechanism result in local spatial filters. This is the consequence of the correlations of the objects are forwarding to the neural network about the external world. It is the competitive process that forces the neurons to find the distinct correlations.

III. Geometry by Hebbian learning

It is an easy task to learn the geometry of the external world with the help of our local filters since in many cases when we present an object to the network, the position of the object is such that more than one neuron shall have non-zero input activities. In this case neurons that assume large input activities have filters that are close to each other, since they are excited by the same object.

In the context of artificial neural networks learning takes place as the development of connections. For this reason we may try to develop connections q_{ij} between neurons i and j according to their input activities:

$$\Delta q_{ij} = \beta(a_i a_j - q_{ij}),$$

where β is the lateral connection learning parameter. The best training results were achieved when only the winning neuron could update its connections:

$$\Delta q_{ij} = \beta(y_i + y_j)(a_i a_j - q_{ij}),$$

where y_k is the output of the k^{th} neuron after competition: the output is 1 for the winning neuron and 0 for the others. In this way $y_i + y_j$ is not zero if and only if either the i^{th} or the j^{th} neuron was winning.

There is one point worth mentioning here, and that is how one can ensure to have a wiring that corresponds to the geometry of the external world. Consider, for example, a field with an elongated lake in the middle. Assume, that objects can move on the field and not on the lake. Two points on the opposite sides of the lake may be very far from each other if one is restricted to the field and cannot pass the the lake. In our view a representation of the geometry is appropriate if there will be no connection between the spatial filters containing those 'distant' points. We want our q_{ij} connections to represent the geometry in the above sense. This may be formulated as follows: We call two receptive fields of two neurons separated, or distant, if there is no input that could overlap with both. The neurons themselves shall also be called separated, or distant neurons. The condition of not developing connections between distant neurons is that neural activity should be zero if the input does not overlap with the neuron's receptive field and vice versa. From now on this condition will be called the *separability condition*. The formulation of that condition and the proof of its necessity may be put on a firm mathematical basis [6]. The problem if this condition is met is hidden in the special form of neural activity function F. It has been shown [6], that the inner product function fulfills the said condition for the 2D problem. In our earlier paper we have shown for the 2D case that local filters close to each other develop connections whereas filters far from each other are not capable of developing connections [7].

In another example we present two 2D projections of 3D objects to two 2D (left and right) 'retinas'. It may be shown, that in this special case the separability condition is fulfilled if the neurons sum up their respective inputs from the two retinas as those are transmitted through the neural filters and then multiply the two sums to determine the neural activities:

$$a_j \sim (\mathbf{w}_j^{(1)}\mathbf{x}^{(1)})(\mathbf{w}_j^{(2)}\mathbf{x}^{(2)})$$

where $\mathbf{w}_j^{(k)}$ $k = 1, 2$ denote the left ($k = 1$) or right ($k = 2$) part of the input filter system and $\mathbf{x}^{(k)}$ $k = 1, 2$ denote the appropriate input vector components. It may be shown that this form of neuron activity fulfills the separability condition, while the previously used inner product function does not [6]. The neural activities are then the subject of competition. The input vector was composed of two

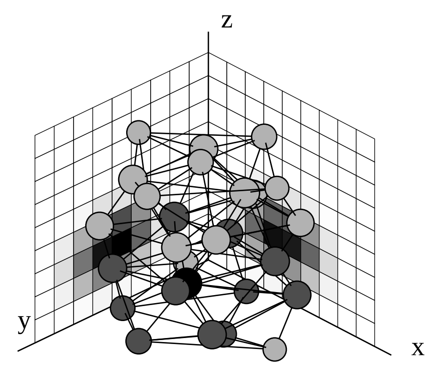

Figure 1: Learnt neigboring relations in 3-dimensions

Two of 2D projections of randomly positioned 3D object were inputted to the neural network. Neurons that could develop connections are linked by lines. One neuron (colored by black) with its receptive fields is shown on the figure. Neighboring neurons – neurons that could develop connections to the black neuron – are colored dark grey. Other neurons are lightgrey. Positions and sizes of circles correspond to the centers and sizes of the neurons' receptive fields, respectively.

144-dimensional vectors and we allowed 27 neurons to develop local filters and to learn the geometry of the external world. If the external world had been a 2D world of 288 pixels then we would now have 18 surviving neurons and 2D wiring. As Fig. 1 shows, all 27 neurons have survived and developed 3D wiring. It means that our network is capable of finding the geometry of the external world just by 'looking at it'.

To show that the geometry of the space is discovered by the network we presented objects in a U-shaped part of the 3D world that could be important for robotic applications where the shortest distance may not be feasible for the robot. The result is shown on Fig. 2. Connection strengths nections. This is not the case, however, as it may be seen comparing Fig. 3 and Fig. 4. In Fig. 4 connection strengths put in an increasing order are shown for the correct and the incorrect neural activities. It may be seen that if the input activity does not satisfy the separability condition then very few neural connections decay to zero as it is expected from breaking the separability condition: there will be inputs from the external world that can simultaneously excite neurons corresponding to separated receptive fields. Thus distant receptive fields develop connections and even thresholding cannot discriminate against these connections. According to Fig. 4 the threshold value 0.45 is very high. This shows clearly, that the correct and incorrect connections are mixed in an inextricable way.

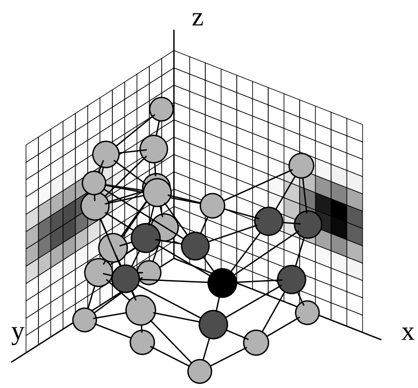

Figure 2: Learnt neighboring relations for a U-shaped volume

were thresholded to cut off the weak connections. As can be seen the connections correspond solely to the possible routes of the world.

If the neural activity is not chosen properly - i.e. the separability condition is not satisfied - then the topology of the external world will not be reflected by the connections between the neurons. To give an example we made runs with the same U-shaped world, but the neural activities were determined simply by summing up all the filtered inputs of the neurons. The resulting connection structure is shown in Fig. 3. The connection structure does not reflect the geometry of the external world. One could try to threshold connections strengths hoping that neurons of neighboring receptive fields has stronger con-

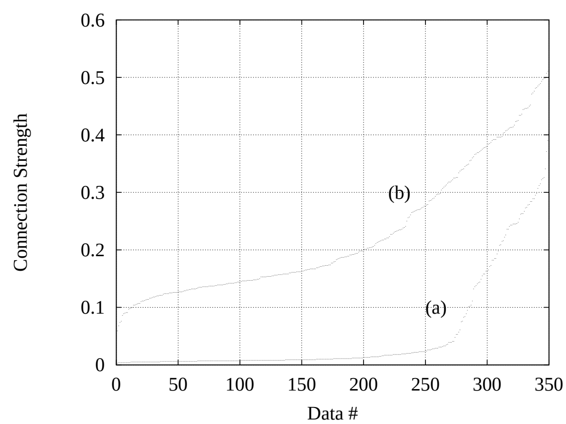

Figure 4: Connection strength in increasing order

Curves (a) and (b) show results of (a) the separability condition is and (b) is not fulfilled, respectively. Connection strength cannot decay to zero for nonconnected parts of the external world, furthermore reach the same level as connection strengths for connected parts.

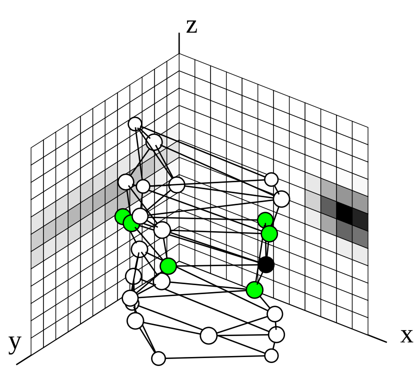

Figure 3: Neighbor learning with neural activities not satisfying the separability condition

Those connections whose strength is above the threshold 0.5 are shown only.

IV. Conclusions

Competitive networks that are capable of creating spatial filters can develop connections that fit the geometry of the external world. These connections are then capable of producing cooperative neighbor training through neural methods [7]. The whole network is self-organizing, and the different self developing structures can develop simultaneously, in other words: the self-organizing processes work together. It is an easy task for the network to discover the 3D world from two 2D orthogonal projections of three dimensional objects.

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